ELASTIC-PLASTIC RESPONSE OF STRUCTURES TO BLAST AND IMPULSE LOADS

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ABSTRACT

This report discusses the general types of failures of typical structures that are used in aircraft and missiles. The theories of elastic and plastic deformation of these structures are presented and comparison with experiments of simple structures is given. It is found that the side on impulse can be used together with rigid-plastic theory of buckled and collapsed cylinders to predict plastic deformation for a wide range of pulse durations. For beam type structures the impulse and rigid plastic approximations seem to only hold for pulses of very short duration.

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LIST OF SYMBOLS

$\bar{E}$	Energy delivered to structure from explosion
V	Work done by internal forces while structure is deforming
$\dot{w}$	Lateral velocity of structure (i.e. velocity perpendicular to surface of structure)
$I(x,y)$	Impulse per unit mass applied to structure
T	Kinetic energy imparted to structure
$\mu(x,y)$	Mass per unit area of structure
I_0	Maximum value of impulse per unit mass
$f(x,y)$	Function describing distribution of impulse over structure
I_t	Total impulse imparted to structure
$\bar{I}$	Impulse per unit area
A	Loaded area of structure
$\delta[U]$	Variation of potential energy
V'	Symbol indicating an integral over the volume
Σ	Symbol indicating an integral over surface
$\bar{A}, \bar{B}, \bar{C}, \bar{D}\ldots$	Constants in the expansion of internal work as a function of maximum lateral deflection
w_0	Maximum lateral deflection
$p(A,t)$	Pressure on the surface of a structure (a function of area location, denoted by A, and time)
$f(A)$	Lateral deflection distribution over the surface of the structure
$\bar{w}_0, \dot{\bar{w}}_0$	Initial values of maximum deflection and velocity
w	Lateral deflection of structure
$w_{0n}(t)$	Contribution of the nth term in the series to the lateral deflection
$f_n(A)$	Function describing the distribution of the deflection in the nth term of the series representing the total lateral deflection of the structure
W	Weight of explosive charge
R	Distance from explosive charge to target
$\bar{E}_f$	Energy flux (energy per unit area) directed toward the target from the explosion
ρ_0	Air density

C_0	Sound velocity
p_0	Amplitude of the exponentially decaying pressure
$\bar{\theta}$	Decay constant of exponentially decaying pressure
$\bar{p}_0$	Ambient pressure
x	Scaled distance (it is actually a dimensionless number equal to $R/.1323\,W^{1/3}$)
E	Modulus of elasticity of structural material
$I_B(x)$	Moment of inertia distribution of beam type structure
$b(x)$	Width of beam structure (a function of x)
$M(\theta)$	Resisting moment at hinge
M_r	Resisting moment for rth segment of the M-θ curve which is approximated by a series of linear segments
K_r, C_r	Constants involved in the representation of the resisting moment in the rth linear segment
θ_r	Hinge angle corresponding to the rth linear segment of the M-θ curve (moment - hinge angle curve)
x_h	Distance from the root section of the beam to the point where the hinge forms
D_x, D_y, H	Stiffness constants of an equivalent orthotropic plate
$w_{ij}(x,y)$	Function describing the amplitude and shape of the ij^{th} term in the double series representing the deflection of the plate
$P(x,y,t)$	Pressure on the plate as a function of space and time
$G(x,y)$	Function representing the spatial distribution of the pressure on the plate
$f(t)$	Function representing the timewise distribution of the pressure on the plate
X_i, Y_j	Beam functions representing the ijth shape of the plate in the x and y directions respectively
β_i, β_j	Frequency numbers associated with the ith and jth beam functions respectively
a	Width of the plate (shorter side)
b	Length of the plate (longer side)
X_i'', Y_j''	Second derivative of the beam functions with respect to x and y respectively

$\sigma_i = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2}$ for biaxial stress systems

$e_i = \sqrt{\epsilon_x^2 + \epsilon_x \epsilon_y + \epsilon_y^2 + \frac{1}{4}\gamma_{xy}^2}$ for biaxial stress systems

$\sigma_x, \sigma_y, \tau_{xy}$ Stresses in plate or shell; σ_x, σ_y are normal stresses; τ_{xy} shear stress

$\epsilon_x, \epsilon_y, \gamma_{xy}$ Strains in plate or shell; ϵ_x, ϵ_y are normal strains; γ_{xy} shear strain

z Distance of any element of plate to neutral plane

λ Parameter describing a linear hardening material; if $\lambda = 0$ the material is elastic; if $\lambda = 1$ the material is perfectly plastic

σ_s, σ_o Yield stress

e_s Yield strain

h Thickness of surface plating in a stiffened plate

σ_u Ultimate stress

T_x, T_y Tensions in x,y directions of an "equivalent thickness" plate

h_x, h_y Equivalent thicknesses of plate in x,y directions respectively

t Thickness of cylindrical shell

a Radius of cylindrical shell

D Diameter of cylindrical shell

L Length of cylindrical shell

$x' = x/L$

x, φ Cylindrical coordinates

d_o Length of hinge line in a collapse pattern

k Parameter describing the decay of the buckled deflection around the periphery of the cylindrical shell

$\bar{a}$ Distance from the edge of a supported shell at which uniform deformation starts in a shell loaded by sprayed explosive

$D = Eh^3/12(1-\nu^2)$

ρ Mass density of shell material

I. Introduction

There are two central aspects involved in calculating the response of structures to blast or other impulsive loading which deforms the structure. The first aspect is the description of the loading and the second is the method of obtaining the elastic and plastic response once the loading is known. The classical method of obtaining the response of any structure is to determine the differential equations governing the behavior of the structure and solve these by some technique such as a finite difference procedure, the Galerkin method, or obtain exact solutions if possible. Such classical solutions are based upon the premise that the loading on the structure is known completely, i.e. that its amplitude and its spatial and temporal characteristics are known. There is much work to be done in determining both the loading and the response to a completely defined load; especially to the point where the analysis can be used with complete confidence in practical situations. It is the purpose of this report to review some of the recent procedures that have been suggested in the past several years for computing the loading and response of beam type, plate type, and shell type structures and to point out some of the advantages and shortcomings by comparing the results with some available experimental results.

II. Damage in structures

A. General considerations

Often in the field of blast damage the terms critical impulse or critical pressure are used to denote values for which the damage reaches a certain approximate value. The concept of critical values is extremely valuable in experimental studies such as those conducted on full scale structures or those conducted by Schuman[1,2,3*] on model shells. However one can and should expect more from a theory than just to predict critical values. In fact due to the inherent nonlinearities in structures the answers that one obtains in the large deflection region is a dependency between load (or some function of load) and deformation.

B. Types of failures

1. Beam type structures (wings, control surfaces, slender missiles, slender fuselages)

In structures which can be replaced by a line with appropriately distributed mass and inertia properties the elastic and plastic behavior is rather straightforward. The structure behaves elas-

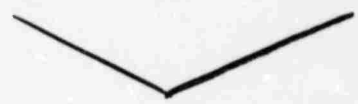
*Superscripts refer to references listed at the end of the report.

tically and responds in its modes until the yield point is reached at some section of the beam. At this section a hinge forms and the structure deforms plastically around this hinge as shown in Figure 1.

1a. Elastic Response of a Cantilever

1b. Elastic Response of a Free-Free Beam

1c. Plastic Response of a Cantilever



1d. Plastic Response of a Free-Free Beam

Fig. 1 Typical Response of a Beam Type Structure

Since these problems are one dimensional and usually involve only one plastic hinge they can be solved by starting with the differential equations of an elastic beam in bending. An investigation was undertaken several years ago[4] to study such problems and a general computer program was developed to solve this general type of problem. This area has been under investigation for a number of years and there have been many papers written on the subject. From a practical standpoint this type of problem is fairly well in hand at the present time if the loading on the structure is adequately defined.

2. Plate type structures (Parts of fuselages, plate type control surfaces, plating between longitudinal and transverse stiffeners)

The flat elastic surface has one more spatial dimension than the beam but its elastic and plastic deformation characteristics are many times more complicated. In the elastic region the governing differential equation usually has to be solved by a two dimensional finite difference network or by some approximate modal method since the exact solution can be obtained only for very few boundary conditions. The classical plasticity problem, especially for the rectangular plate, offers an extremely difficult problem even for very small plastic deformations. For that reason approximate approaches were developed several years ago[5,6] for small and large elastic and plastic deformations of plates.

Cylindrical Shells

The elastic analysis of cylindrical shells has been the subject of many studies in recent years. However the only plastic problems associated with cylindrical shells that have been approached from the classical viewpoint of flow plasticity is the axially symmetric problem.[7] Other problems have been considered from the standpoint of deformation theory.[8] The real problems in blast deformation of cylindrical conical and other types of shells or composite bodies are not axisymmetric and involve situations where the pressure time relation on the shell is not easily obtainable. This is the reason why a work-energy approach has been followed in several recent reports.[9,10] In that work the asymmetry of the load and deformation was considered as well as strain hardening, large nonlinear deformations, and thermal aspects. It was shown experimentally by Schuman[1,2,3] that cylindrical shells under a blast which engulfs the structure either fails in a well defined buckling pattern or a collapse pattern. Moreover, for loading which is more of a local type such as sprayed explosive the deformation was almost uniform along the length but changed rather abruptly to zero at the supported edges. In addition if the explosive was sprayed over half the periphery, then only this much deformed. A representative pattern is shown in Fig. 2.

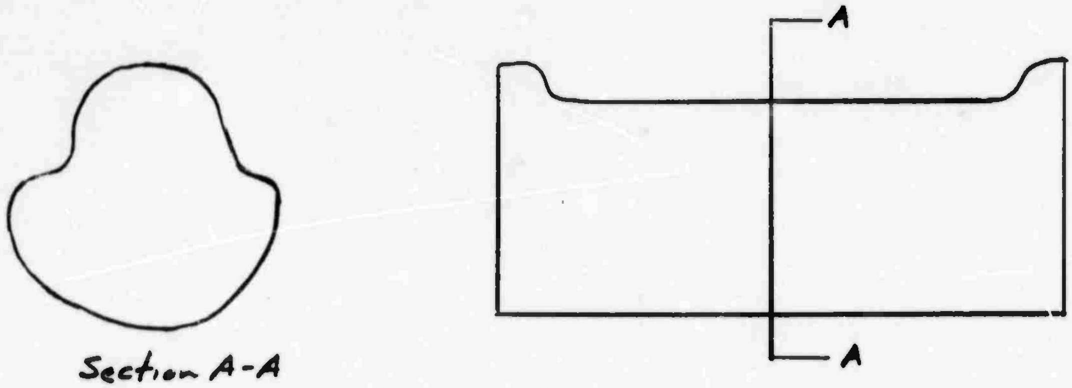


Fig. 2 Typical Deformation Pattern from Sprayed Explosive Over Half of a Fixed-Fixed Shell

Deformation of composite structures

Actual aircraft and missiles are composite structures which can be considered, in an approximate sense, to be made up of beam, plate, and shell components. In such structures several types of failure may occur simultaneously, and for this reason careful judgement should be exercised in evaluating the blast resistant characteristics of such built-up structures. For example, an aircraft is composed schematically of a beam type fuselage which is built-up of a shell with ring and longitudinal stiffeners. The wings can be considered as beams with variable mass and stiffness. The fuselage can be considered as a shell or a beam when ex-

posed to an engulfing blast which loads the entire structure but it can also be considered as a series of stiffened and unstiffened panels. By the same token, the wings, elevator and stabilizer act either as beams or a series of panels.

In general we can divide the deformation into primary, secondary and tertiary components as was done for elastic deflection of ships a number of years ago.[11] The primary deformation consists of deflection of the entire structure (wing or fuselage) as a beam. The secondary deformation involves deflection of entire stiffened plate or shell sections and the tertiary deformation involves deflection of plating between stiffeners or shell deformation between rings.

It is important to look critically at each type of failure to see if it would kill the utility of the structure in performing its mission. Referring to Fig. 3 it is seen that deformation can take place in the following ways:

1. Overall hinging of the wing, rudder and stabilizer
2. Deformation of the panels contained in the fuselage, wing and tail assembly
3. Buckling or failure of a section of the fuselage and consequent hinging of the fuselage around the weakened buckled section

In a missile type structure shown in the same figure the deformation patterns are similar.

Lumped parameter vs. classical theories

In classical theory the differential equations of the structure are set up for the elastic and plastic ranges and the appropriate pressure distribution in space and time is used. The resulting answers are deflections, stresses, and strains as a function of time at many locations on the structure. This type of theory for predicting elastic and plastic behavior is practical for beam type structures where only one deflection and only one space dimension is present. For plate and shell structures where two space dimensions are present and more than one deflection may be necessary to describe the behavior of the structure, the analysis is many times more involved. For many cases such as large lateral deformations of plates and cylindrical shells an equivalent energy procedure and variational approach such as used in previous reports[10] offers many simplifications. In the equivalent energy approach the estimated energy delivered from the explosion is equated to the internal work done by the structure in plastically deforming. The main unknown which has to be inserted into the theory is the deflection pattern in the plastic region, which is determined from tests. Once this deformation shape is defined, it is straight forward to compute the work done by internal forces in the structure to deform plastically in this pattern. In many cases it is the maximum damage or plastic deformation which is of primary significance. The lumped parameter or energy type analysis is capable of predicting the necessary informaton for many situations.

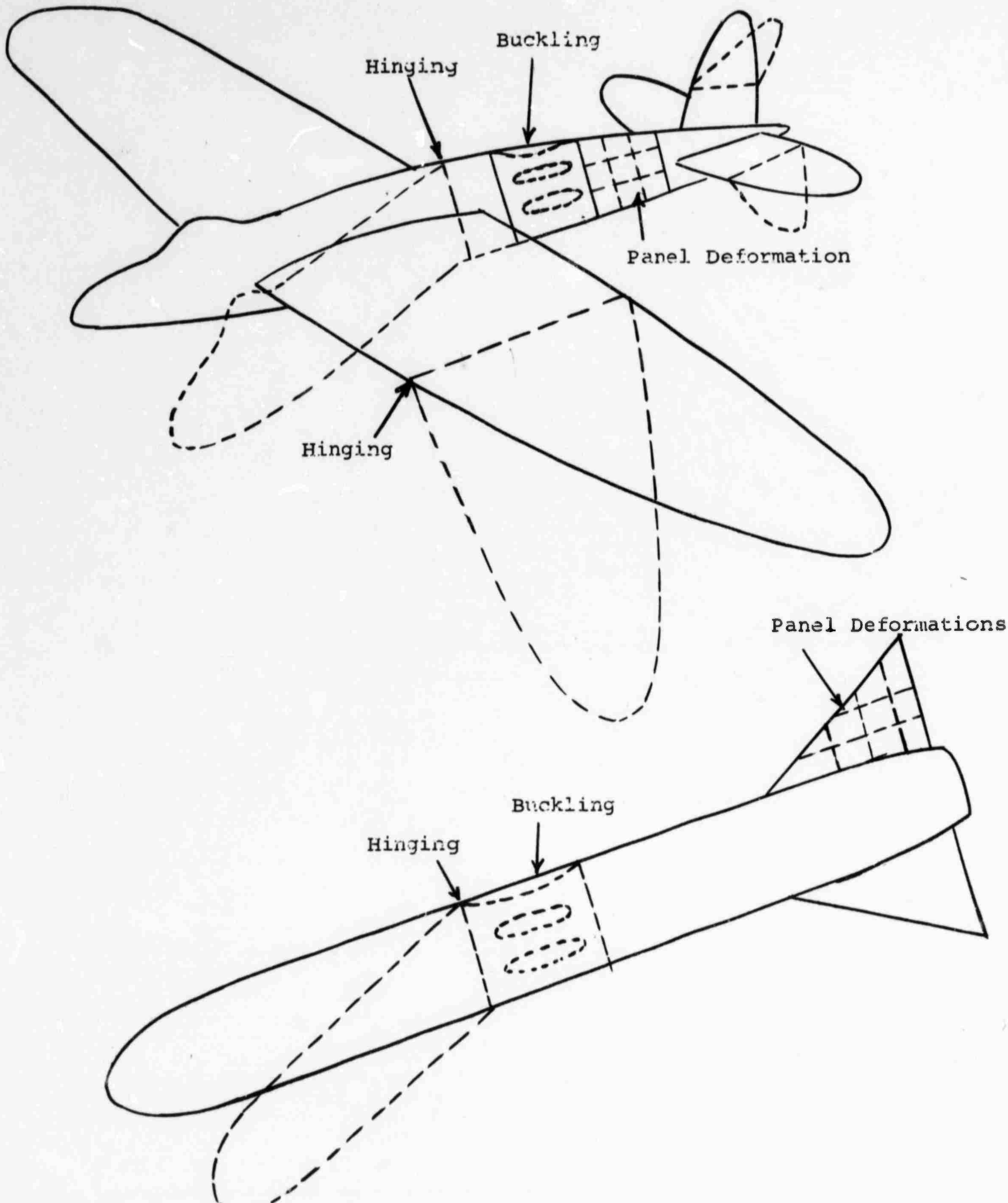


Fig. 3 Schematic of Various Types of Failure

It will be seen later that for short pulses the above impulse or energy approach is accurate, but for some cases involving longer duration blasts we have to make recourse to a more accurate procedure which uses the spatial and time characteristics of the load. The analysis which fits this qualification is the variational approach. In this approach a variational principle associated with deformation type (as opposed to flow type) plasticity is used; the principle is essentially Hamilton's principle interpreted for the plastic region.

IV. Fundamental energy-impulse relations and the variational principle

A. Energy - impulse

Let V represent the work done by the internal forces while the structure is deforming and let $\bar{E}$ represent the total energy delivered to the structure from the explosion; then the fundamental energy relation that will be used is as follows:

$$\bar{E} = V \qquad [1]$$

For cases where the impulse is given or can be obtained more easily than the energy, an alternate relation is derived below.

Let I be the impulse per unit mass applied to the structure. The impulse momentum relation for an elemental mass dm can be written (neglecting the tangential and longitudinal velocity, assuming that the lateral velocity, $\dot{w}$ is much greater than the other two)

$$\dot{w}\, dm = I\, dm$$

Thus

$$\dot{w} = I \qquad [2]$$

The Kinetic energy imported to the structure is

$$T = \int_A \tfrac{1}{2} \mu(x,y)\, \dot{w}^2 dA = \tfrac{1}{2}\int_A \mu(x,y)\, I^2 dA \qquad [3]$$

where $\mu(x,y)$ is the mass per unit area of the structure (which can vary) and dA is an elemental area. The impulse can vary over the surface so write it as follows:

$$I(x,y) = I_0 f(x,y) \qquad [4]$$

Thus

$$T = \frac{I_0^2}{2}\int_A \mu(x,y) f^2(x,y)\, dA \qquad [5]$$

Equating this initial Kinetic energy to the energy of deformation absorbed by the structure the expression for the impulse per unit mass becomes

$$I_0 = \sqrt{\frac{2V}{\int_A \mu(x,y) f^2(x,y)\, dA}} \qquad [6]$$

The total impulse on the structure will then be

$$I_t = \int_A \mu(x,y)\, I_0 f(x,y)\, dA = \int_A \mu(x,y) \sqrt{\frac{2V}{\int_A \mu(x,y) f^2(x,y)\, dA}}\, f(x,y)\, dA \qquad [7]$$

The impulse per unit area will be

$$\bar{I}(x,y) = I_0 \mu(x,y) = \mu(x,y) \sqrt{\frac{2V}{\int_A \mu(x,y) f^2(x,y)\, dA}} \qquad [8]$$

If both the mass per unit area and the impulse are uniform then

$$\bar{I} = \sqrt{\frac{2\mu V}{A}} \qquad [9]$$

where A is the total loaded area of the structure.

B. The variational approach

The principle of extremum potential energy can be applied to the plastic regime if we are dealing with a plastic deformation theory (i.e. not a flow theory) as we are in this work. Greenberg[12] states this principle as follows:

$$\delta [U] = 0 \qquad [10]$$

where
$$U = \int_{V'} \left(\int_0^{\epsilon_i} \sigma_i d\epsilon_i \right) dV' - \int_\Sigma T_i u_i \, d\Sigma = V - W$$

The integral over V' is the potential energy or work done by the internal forces during deformation and the integral over Σ is the work done by the external surface forces which give rise to the internal forces.

The dynamic counterpart of the principle of extremum potential energy is Hamilton's Principle[13] which can be stated as follows:

$$\delta \int_{t_1}^{t_2} [T - U]\, dt = 0 \qquad [11]$$

where T is the Kinetic energy and V is the potential energy minus the work done by the external forces.

We are dealing with large lateral deformations of thin walled structures where we know the deformation pattern from test results but the amplitude of lateral deformation, w, is inknown. If we neglect the longitudinal and tangential inertia terms the Kinetic energy of the structure becomes

$$T = \frac{1}{2} \int_A \mu(A)\, \dot{w}^2\, dA \qquad [12]$$

μ = mass per unit area of the structure

dA = element of surface area

$\dot{w}$ = velocity normal to surface of structure

= $\dot{w}_0(t) f(A)$ where $w_0(t)$ denotes the time dependency of the deflection and $f(A)$ denotes the space dependency

The variation is taken just as in the elastic problem as given in Love.[13]

This problem is equivalent to the following problem in the calculus of variations:

Find the function y(x) which takes on a given value for x = a and b and which minimizes the definite integral

$$I' = \int_a^b F(x, y, y')\,dx \qquad [13]$$

The result is that F must satisfy the Euler equation

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0 \qquad [14]$$

In our case

$$F = T - U$$
$$y' \to \dot{w}_o(t) \quad , \quad x \to t \quad , \quad y \to w_o(t) \qquad [15]$$

Thus we have

$$T = \frac{1}{2}\int_A \mu(A)\,\dot{w}^2 dA = \frac{1}{2}\int_A \mu(A)\,\dot{w}_o^2(t)\,f^2(A)\,dA \qquad [16]$$

In general we can fit a power series to the potential energy (or work done by the internal forces)
i.e.

$$V = \bar{A} + \bar{B}w_o + \bar{C}w_o^2 + \bar{D}w_o^3 + \cdots \qquad [17]$$

$$W = \int_A p(A,t)\,w_o\,f(A)\,dA \qquad [18]$$

where $p(A,t)$ is the pressure applied to the surface of the structure

The result is the following nonlinear differential equation in time for the unknown w_o

$$\ddot{w}_o \int_A \mu(A)\,f^2(A)\,dA + \left[\bar{B} + 2\bar{C}w_o + 3\bar{D}w_o^2 + \cdots\right] = \int_A p(A,t)\,f(A)\,dA \qquad [19]$$

The initial conditions are

$$w_o(0) = \bar{w}_o \qquad , \quad \dot{w}_o(0) = \dot{\bar{w}}_o \qquad [20]$$

where $\bar{w}_o$ and $\dot{\bar{w}}_o$ are the deflection and velocity at the beginning of the plastic regime which are determined from the elastic analysis when the yield point is reached.

Instead of the deflection being a function of one particular shape supposing

$$w = \sum_n w_{on}(t)\,f_n(A) \qquad [21]$$

then

$$T = \frac{1}{2}\int_A \mu(A)\,\dot{w}^2 dA = \frac{1}{2}\int_A \mu(A)\left[\sum_n \dot{w}_{on}(t)\,f_n(A)\right]^2 dA$$
$$V = V(w, w^2, w^3, \text{etc.}) \qquad [22]$$

Unless we assume complete decoupling among the terms the problem will get completely out of hand; thus we will assume that this problem reduces to finding the set of w_{on}'s satisfying

$$\ddot{w}_{0n}\int_A m(A)\, f_n^2(A)\, dA + \left[B_n + 2C_n w_{0n} + 3D_n w_{0n}^3 + \cdots\right] = \int_A p(A,t)\, f_n(A)\, dA \qquad [23]$$

with initial conditions on displacement and velocity determined from the end of the elastic region.

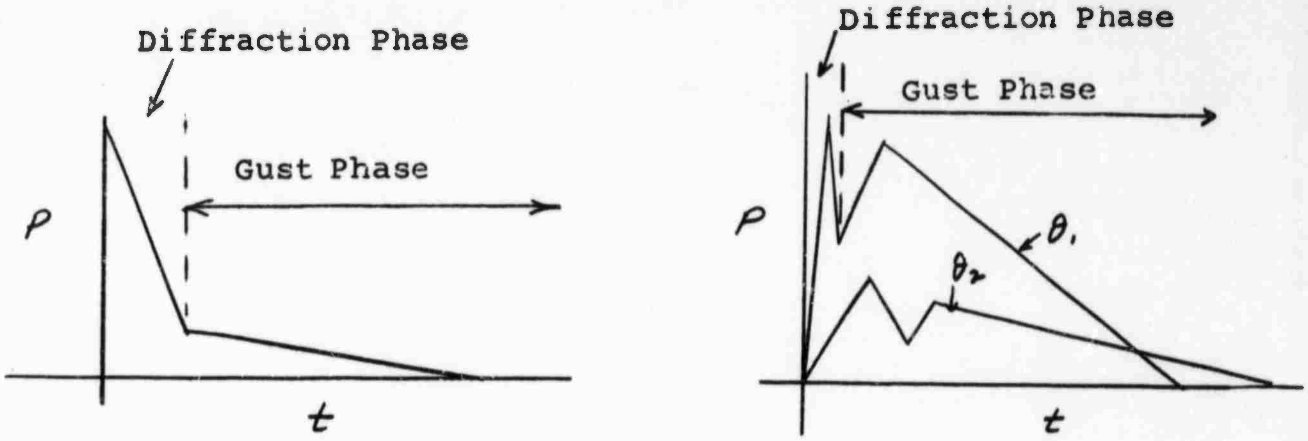
V. Description of the parameters associated with the loading

A. General problem

Since the structure reacts with the blast it is usually very difficult to determine the complete spatial and temporal characteristics of the pressure which acts on the structure. When the blast comes in contact with the structure it diffracts (bends) around the structure in a very short time and then produces a drag type loading which lasts a much longer period. In general even the deformation of the structure will effect the pressure distribution and in many cases involving control surfaces subject to blast this is considered. In many of the more complicated cases it is possible to use some parameter to represent the load other than the pressure distribution itself, such as energy or impulse. In such cases the structural theory must be altered accordingly in order to compute characteristics of the system, such as energy absorption, which are most easily integrated with this load parameter.

B. Pressure distributions

There are a limited number of cases where pressure as a function of space and time has been obtained for rigid bodies of various shape. The Nuclear Effects Handbook[14] gives a particularly good outline of the results to date. A recent book by Kornhauser[15] also gives pressure curves for some particular cases. For aircraft and missile structures we can approximate the pressure distribution and time history by representing the shapes of the various components of the structure by idealized shapes such as long slender boxes and cylinders and then use the pressure distribution for these shapes. The wings and other control surfaces of aircraft and missiles can be represented by flat beam type surfaces (long slender boxes) and the fuselages can be represented by cylindrical shapes. The problem is one of finding the loading while the weapons are parked and when they are in the air. The pressure time history for a blast on a parked vehicle is shown schematically in Fig. 4.[14]



Prsssure for Lifting Surfaces

Pressure for Fuselages (Different plot for each angle around cylinder)

Fig. 4 Schematic of Pressure Time History for Parked Vehicles and Non-Lifting Bodies Subject to Blast

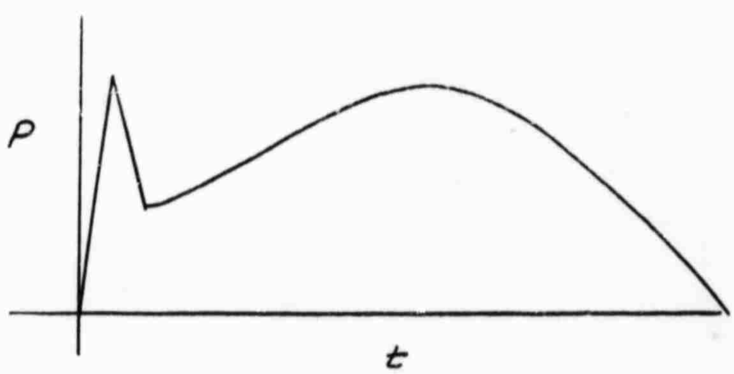


Fig. 5 Schematic of Pressure Time History for Lifting Surfaces in Flight Subject to Blast

The various parameters associated with each of these plots can be obtained from the above mentioned reference[14] and work by Baker et. al.[16] and Baker and Schuman.[17] Extensive data on incident and reflected pressure and time duration of pressures associated with explosions of pentolite is contained in reports by Baker, et. al.[16] and Goodman.[18] The use of this data will be illustrated later in the report when some specific problems are considered.

During flight the problem has to be handled as a lifting body gust problem and typical pressure time histories would look schematically as shown in Fig. 5[19,20] for lifting surfaces. Although there is some lift on the fuselage it is believed that the parked condition can be used as a first approximation.

C. Energy and impulse of an engulfing explosion

1. Energy

For blasts which are at some distance from the structure the blast wave engulfs the entire target when it reaches it. The characteristics of the blast are described by a charge weight (or weight of explosive) and a distance from the target. The weights for nuclear explosions are given in terms of equivalent weight of TNT.[14] For explosions involving pentolite instead of TNT the conversion is that 1.1 pound of TNT is equivalent to 1 pound of pentolite and the weight of pentolite in free air is equivalent to 1.8 times its weights on the ground.[17] For underwater explosions experiments have shown[21] that the energy flux density (energy per unit area) and the impulse per unit area before the shock wave hits the structure can be written functionally as follows:

$$\bar{I} = C W^{\frac{1}{3}} \left(\frac{W^{\frac{1}{3}}}{R}\right)^{\beta}, \quad \bar{E}_f = k W^{\frac{1}{3}} \left(\frac{W^{\frac{1}{3}}}{R}\right)^{\gamma} \qquad [24]$$

where C, k, β and γ are constants which are determined by fitting experimental results to the above relations.[21] Using the work of Baker and Schuman[17] a similar analysis can be made on air blast of pentolite explosive as will be shown below.

Following the reasoning of Keil,[22] if it is assumed that the blast wave is plane then the energy flux (energy per unit area) can be written

$$\bar{E}_f = \frac{1}{\rho_o C_o} \int_0^{\infty} p^2(t)\, dt \qquad [25]$$

where ρ_o is the air density, C_o is the sound velocity in the air and p(t) is the pressure at any given point as a function of time. If it is further assumed that the wave is exponentially decaying, then

$$p(t) = p_o e^{-t/\bar{\theta}} \qquad [26]$$

where p_o is the pressure amplitude at t = 0 and $\bar{\theta}$ is the time constant of the decay. Using the above two relations it is found that

$$\bar{E}_f = \frac{p_o^2 \bar{\theta}}{\rho_o C_o} \qquad [27]$$

The experimental curves of Baker and Schuman[17] can then be used to obtain p_o and $\bar{\theta}$ as a function of charge weight and distance. Substituting these extrapolated values of p_o and $\bar{\theta}$ and using the appropriate value for ρ_o and C_o the following final relation is found for the energy flux (energy per unit area) in the blast field at a distance R from the center of the explosion:

$$\bar{E}_f \approx 19.2 \times 10^3 \left(\frac{W^{5/4}}{R^{11/4}} \right) \frac{\# \text{ in}}{\text{in}^2} \quad [28]$$

where R must be given in feet and W in #. It is unfortunate that we have to attach specific dimensions to the input and that we have to mix inches and feet in the same formula. However, it is most convenient to present the relation in this form since it is empirical and was obtained by fitting the relation to data that was given originally in these specific dimensions.

2. Impulse

Values of side on (i.e. incident) impulse and reflected impulse (i.e. reflected from a flat rigid target of infinite extent) were studied extensively by Goodman[18] for free air blast of bare spherical pentolite. Goodman's formulas are as follows:

Side on: $$\frac{\bar{I}}{\bar{p}_o^{2/3} W^{1/3}} = \left[\frac{478.7}{p_o^{1/3} X} - \frac{3593}{(p_o^{1/3} X)^2} \right] \frac{\text{psi millisec}}{\text{lb}^{1/3}} \quad [29]$$

Reflected: $$\frac{\bar{I}}{\bar{p}_o^{2/3} W^{1/3}} = \left[\frac{910.2}{p_o^{1/3} X} + \frac{7908}{(p_o^{1/3} X)^2} + \frac{1601}{(p_o^{1/3} X)^3} \right] \frac{\text{psi millisec}}{\text{lb}^{1/3}} \quad [30]$$

where $\bar{I}$ is the impulse per unit area (in psi milliseconds) p_o is the ambient sea level pressure in atmospheres, W is the charge weight in pounds and X is a dimensionless number equal to

$$X = R / .1323 W^{1/3} \quad [31]$$

in which R is the distance from the center of the explosion to the target.

D. Impulse and energy content for local explosion (such as sprayed explosive)

As was seen above, for explosions that are at some distance from the target the weight of explosive and distance away determines the energy content at any point in the field. For impulsive loading or any type of explosion which is an integral part of the structure such as sprayed explosive, the problem is not so straight forward. In these cases it will be assumed that the impulse per unit area, $I(x,y)$ applied to the structure can be measured. Then the impulse relation in Section IVA can be employed to determine the deformation, i.e.

$$I(x,y) = \mu(x,y) \sqrt{\frac{2V}{\int_A \mu(x,y) f^2(x,y) \, dA}} \quad [32]$$

in which μ is the mass per unit area, V is the energy absorbed and $f(x,y)$ describes the impulse distribution. If the impulse and mass per unit area are both uniform then the energy flux will be

$$\bar{E}_f = \frac{\bar{E}}{A} = \frac{V}{A} = \frac{1}{2}\frac{(\bar{I})^2}{\mu} \qquad [33]$$

VI. Work done by internal forces and coupling with the variational principle

A. Beam type structures

1. General

The complete theory for hinging deformation of beam type lifting surfaces was derived by the MIT group[23,24] and by the present author.[4,6] In the event that complete load-time histories are available it is certainly preferable to use the complete beam type theories mentioned above. However for rough calculations and in cases where overall damage criteria are being sought a theory consistent with the impulse or energy approach and the variational approach discussed previously can be employed.

2. Elastic region

Before plastic hinging occurs, it is assumed that the beam vibrates in its elastic modes. The kinetic and potehtial energy of the structure in the elastic region can then be written as follows:

$$\text{KE} \qquad T = \frac{1}{2}\int_0^l \mu(x)\left[\dot{w}(x,t)\right]^2 dx \qquad [34]$$

$$\text{PE} \qquad V = \frac{1}{2}\int_0^l EI_B(x)\left[\frac{\partial^2 w(x,t)}{\partial x^2}\right]^2 dx$$

Assuming a deflection of the form

$$w = \sum_n w_{on}(t)\, f_n(x) \qquad [35]$$

we then have (noting that cross product terms disappear due to orthogonality of modes)

$$T = \sum_n \frac{\dot{w}_{on}^2}{2}\int_0^l \mu(x)\, f_n^2(x)\, dx \qquad [36]$$

$$V = \sum_n \frac{w_{on}^2}{2}\int_0^l EI_B(x)\left(\frac{\partial^2 f_n}{\partial x^2}\right)^2 dx$$

Substituting into Hamilton's principle (or into Lagrange's equations) we obtain the following differential equation for $w_{on}(t)$

$$\ddot{w}_{on}\left[\int_0^l \mu(x) f_n^2(x)\,dx\right] + w_{on}\left[\int_0^l EI_B(x)\left(\frac{\partial^2 f_n}{\partial x^2}\right)^2 dx\right] = \int_0^l p(x,t)\, f_n(x)\, b(x)\, dx \qquad [37]$$

where $\mu(x)$ is the mass per unit length, $p(x,t)$ is the pressure, $b(x)$ is the local width of the beam. The initial conditions under the assumption of blast loading are

$$w_{on}(0) = \dot{w}_{on}(0) = 0 \qquad [38]$$

3. Plastic region

In the plastic region the potential energy is replaced by the work done at the hinge as follows:

$$V = \int_{\theta_1}^{\theta_2} M(\theta)\,d\theta \qquad [39]$$

where $M(\theta)$ is the resisting moment at the hinge. The resisting moment is a function of θ and for most of the practical cases it will take one of the following forms shown in Fig. 6.[6] It is seen that the strain hardening and unstable cases are approximated by a series of linear segments. Thus the most general expression for the hinge resisting moment, M, in each segment of the M-θ curve can be written

$$M_r = K_r\theta + C_r \quad \begin{cases} K_r(+) \text{ for strain hardening} \\ K_r = 0,\ C_r = M_0 \text{ for perfectly plastic} \\ K_r(-) \text{ for unstable} \end{cases} \qquad [40]$$

Thus

$$V = \sum_r \int_{\theta_{r-1}}^{\theta_r} M(\theta)\,d\theta = \sum_r \int_{\theta_{r-1}}^{\theta_r} (K_r\theta + C_r)\,d\theta = \sum_r \left[\frac{K_r\theta^2}{2} + C_r\theta\right]_{\theta_{r-1}}^{\theta_r} \qquad [41]$$

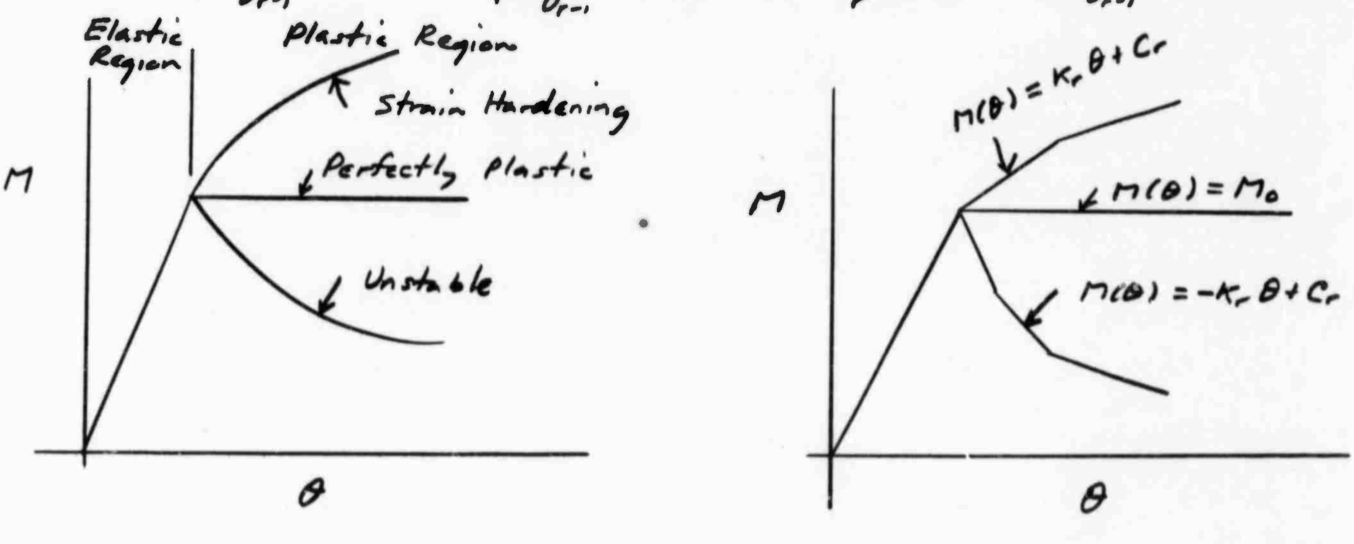


Fig. 6 Schematic of Hinge Resisting Moment Curves

The kinetic energy can be written

$$T = \frac{1}{2}\int_0^{\ell} \mu(x)\left[(x - x_\ell)\dot{\theta}\right]^2 dx \qquad [42]$$

where x_ℓ is the distance from the root section of the beam to the point where the hinge forms.

Thus for the values of θ between θ_{r-1} and θ_r the equation of motion becomes

$$\ddot{\theta}\left[\int_0^{\ell} \mu(x)(x - x_\ell)^2 dx\right] + K_r\theta + C_r = \int_0^{\ell} p(x,t)\,b(x)(x - x_\ell)\,dx \qquad [43]$$

Thus we have linear differential equations in both the elastic and plastic regions. The one in the plastic region has to be solved piecewise for each segment.

An altermate method of dealing with the plastic region is to represent the M- θ curve by a power series in θ as follows:

$$M = C_0 + C_1\theta + C_2\theta^2 + \cdots \tag{44}$$

The resulting differential equation is then the following nonlinear equation

$$\ddot{\theta}\left[\int_0^{\ell}\mu(x)(x-x_A)^2 dx\right] + C_1 + 2C_2\theta + 3C_3\theta^2 + \cdots = \int_0^{\ell} p(x,t)b(x)(x-x_A)dx \tag{45}$$

The initial conditions for the plastic region are the displacement and velocity at the end of the elastic region. If we use the linear segmented curve then the initial conditions for each segment are obtained from the final conditions of the previous segment. Using the Allen interpretation of the "frozen hinge"[25] it is assumed that the final plastic deformation of the structure consists of a rotation around the hinge and occurs at the time when the angular velocity reaches zero for the first time.

B. Plate type structures

1. Elastic deformation

For unstiffened or stiffened and sandwich plates the elastic behavior of the plate can be computed by using the theory developed in a paper by the author, published several years ago.[5] The basic equations governing the behavior of the plate is the orthotropic plate equation

$$D_x\frac{\partial^4 w}{\partial x^4} + 2H\frac{\partial^4 w}{\partial x^2\partial y^2} + D_y\frac{\partial^4 w}{\partial y^4} + \mu\frac{\partial^2 w}{\partial t^2} = P(x,y,t) \tag{46}$$

In the above mentioned paper the solution is given in detail for a rectangular plate under dynamic loading. This elastic solution can be written as follows:

$$w = \sum_i\sum_j w_{ij}(x,y)R_{ij}(t) \tag{47}$$

where

$$w_{ij}(x,y) = X_i Y_j \frac{\int_0^a\int_0^b G(x,y)X_i Y_j\,dx\,dy}{p_{ij}^2\mu\int_0^a\int_0^b X_i^2 Y_j^2\,dx\,dy} \tag{48}$$

$$R_{ij}(t) = p_{ij}\int_0^t f(\tau)\sin p_{ij}(t-\tau)\,d\tau \tag{49}$$

$$p_{ij} = \left\{\frac{D_x}{\mu}\frac{\beta_i^4}{a^4} + \frac{D_y}{\mu}\frac{\beta_j^4}{b^4} + \frac{2H\int_0^a\int_0^b X_i X_i'' Y_j Y_j''\,dx\,dy}{\mu\int_0^a\int_0^b X_i^2 Y_j^2\,dx\,dy}\right\}^{1/2} \tag{50}$$

In the above equations D_x, D_y and H are the orthotropic constants which can be determined from the analysis given in an earlier paper,[26] μ is the mass per unit area of the plate, p_{ij} is the natural frequency of the ij^{th} mode, X_i and Y_j are beam functions describing the modal pattern in the x and y directions respectively, β_i and β_j are constants associated with these mode patterns, a is the shorter side of the plate (parallel to the x axis), b is the longer side of the plate (parallel to the y axis), $G(x,y)$ describes the spatial distribution of the load and f(t) describes the timewise distribution.

2. Plastic deformation

The theory of plastic deformation of isotropic and stiffened plates is given in a recent report.[27] The work of deformation, V per unit volume of an elastic-plastic body can be written

$$V = \int_0^{e_i} \sigma_i \, de_i + \frac{K_v \theta_v^{2}}{2} \qquad [51]$$

where

$$\sigma_i = \frac{\sqrt{2}}{2}\sqrt{(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)}$$

$$e_i = \frac{\sqrt{2}}{3}\sqrt{(\epsilon_x-\epsilon_y)^2+(\epsilon_y-\epsilon_z)^2+(\epsilon_z-\epsilon_x)^2+\frac{3}{2}(\gamma_{xy}^2+\gamma_{yz}^2+\gamma_{zx}^2)}$$

$$\theta_v = \epsilon_x+\epsilon_y+\epsilon_z$$

The relation between σ_i and e_i describes the stress strain law of the material. Assuming an incompressible material ($\theta = 0$) and considering only plate stresses,

$$V = \int_{V_0}\left[\int_0^{e_i}\sigma_i \, de_i\right] dV_0 \qquad [52]$$

where dV_0 = element of volume

$$\sigma_i = \sqrt{\sigma_x^2-\sigma_x\sigma_y+\sigma_y^2+3\tau_{xy}^2} \qquad [53]$$

$$e_i = \sqrt{\epsilon_x^2+\epsilon_x\epsilon_y+\epsilon_y^2+\frac{1}{4}\gamma_{xy}^2}$$

If we consider that only the lateral deformation, w, is of significance and neglect the longitudinal deflection u and the transverse deflection v then the strains ϵ_x, ϵ_y and γ_{xy} become[28]

$$\epsilon_x = \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - z\frac{\partial^2 w}{\partial x^2} \qquad \epsilon_y = \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - z\frac{\partial^2 w}{\partial y^2} \qquad [54]$$

$$\gamma_{xy} = \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} - 2z\frac{\partial^2 w}{\partial x \partial y}$$

where z is the distance of any element of the plate from the neutral plane as shown in the figure below:

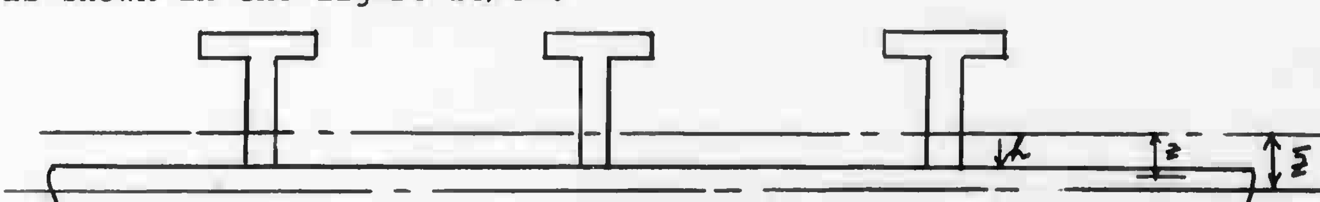


Fig. 7 Location of Plate Element

Further restrict the material to one that obeys an elastic-linear hardening law as shown below

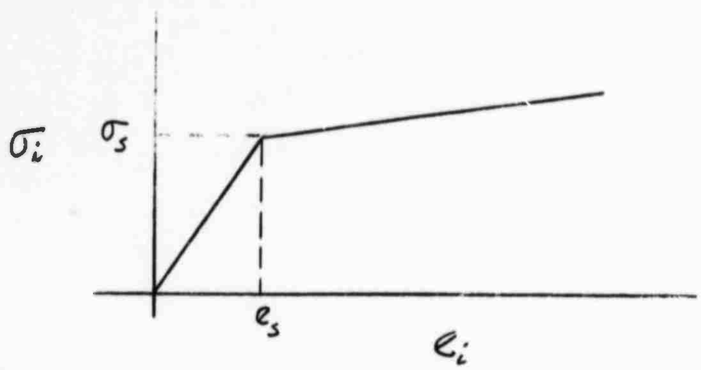


Fig. 8 Linear Hardening Stress-Strain Curve

The stresses can be written in terms of strains as follows:

$$\sigma_x = \frac{4}{3}\frac{\sigma_i}{e_i}\left(\epsilon_x + \frac{1}{2}\epsilon_y\right) \qquad [55]$$

$$\sigma_y = \frac{4}{3}\frac{\sigma_i}{e_i}\left(\epsilon_y + \frac{1}{2}\epsilon_x\right)$$

$$\tau_{xy} = \frac{1}{3}\frac{\sigma_i}{e_i}\gamma_{xy}$$

in which the stress-strain law can be written

$$\frac{\sigma_i}{e_i} = E\left[1 - \omega(e_i)\right] \qquad [56]$$

where $\omega(e_i) = 0 \quad \text{for } e_i < e_s \quad (\text{elastic})$

$$\omega(e_i) = \lambda\left(1 - e_s/e_i\right) \quad (\text{plastic}) \qquad \lambda = 1 - \frac{1}{E}\frac{d\sigma_i}{de_i}$$

Substituting the linear hardening law into the expressions for the stresses and then into the relation for V, we obtain the work done by the internal forces on the plating of the stiffened plate

$$V_1 = \int_0^a\int_0^b\int_{\bar{z}-\frac{h}{2}}^{\bar{z}+\frac{h}{2}}\left[\frac{E e_i^2}{2}(1-\lambda) + E\lambda e_s e_i\right]dx\,dy\,dz - \int_0^a\int_0^b\int_{\bar{z}-\frac{h}{2}}^{\bar{z}+\frac{h}{2}}\frac{E\lambda e_s^2}{2}\,dx\,dy\,dz \qquad [57]$$

where $\bar{z}$ is the distance from the neutral plane of the stiffened plate to the midsurface of the face, h is the thickness of the plate and a,b and the width and length of the plate respectively. If there is a top plate then there is a similar expression V_2 that has to be added for this part, i.e.

$$V_2 = \int_0^a\int_0^b\int_{\bar{z}-\frac{h}{2}}^{\bar{z}+\frac{h}{2}}\left\{\text{Same expression as in } V_1\right\} \qquad [58]$$

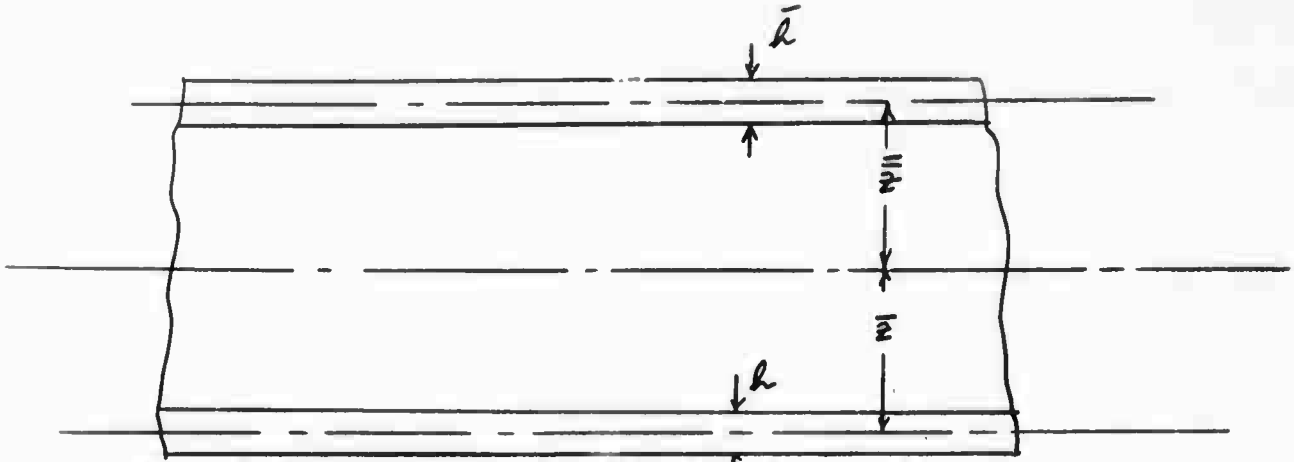


Fig. 9 Nomenclature for Top and Bottom Plates

Now substituting the expression for e_i

$$V_1 = \int_0^a\int_0^b\int_{\bar{z}-\frac{h}{2}}^{\bar{z}+\frac{h}{2}}\Big[\frac{E}{2}(1-\lambda)\,\frac{4}{3}\left(\epsilon_x{}^2+\epsilon_x\epsilon_y+\epsilon_y{}^2+\frac{1}{4}\gamma_{xy}{}^2\right) + \frac{E\lambda e_s}{\sqrt{3}}\,2\sqrt{\epsilon_x{}^2+\epsilon_x\epsilon_y+\epsilon_y{}^2+\frac{1}{4}\gamma_{xy}{}^2}\,\Big]\,dx\,dy\,dz - \int_0^a\int_0^b\int_{\bar{z}-\frac{h}{2}}^{\bar{z}+\frac{h}{2}}\frac{E\lambda e_s{}^2}{2}\,dx\,dy\,dz \qquad [59]$$

Substituting the expressions for the strains in terms of deflections, we have

$$V_1 = \int_0^a\int_0^b\Big\{\frac{E(1-\lambda)}{2(1-\nu^2)}\left(\alpha z+\gamma\frac{z^2}{2}+\beta\frac{z^3}{3}\right)\Big|_{z=\bar{z}-\frac{h}{2}}^{z=\bar{z}+\frac{h}{2}} + \frac{2E\lambda e_s}{\sqrt{3}}\left[\frac{(2\beta z+\gamma)\sqrt{\alpha+z\gamma+z^2\beta}}{4\beta}+\frac{4\alpha\beta-\gamma^2}{8\beta\sqrt{\beta}}\sinh^{-1}\left(\frac{2\beta z+\gamma}{\sqrt{4\alpha\beta-\gamma^2}}\right)\right]_{z=\bar{z}-\frac{h}{2}}^{z=\bar{z}+\frac{h}{2}}\Big\}\,dx\,dy - \int_0^a\int_0^b\frac{E\lambda e_s{}^2}{2}\,h\,dx\,dy \qquad [60]$$

and a similar expression for V_2 if two surface plates exist.

In the above equation

$$\alpha = \frac{1}{4}\left(\frac{\partial w}{\partial x}\right)^4+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\left(\frac{\partial w}{\partial y}\right)^2+\frac{1}{4}\left(\frac{\partial w}{\partial y}\right)^2$$

$$\beta = \left(\frac{\partial^2 w}{\partial x^2}\right)^2+\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right)+\left(\frac{\partial^2 w}{\partial y^2}\right)^2+\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2 \qquad [61]$$

$$\gamma = -\left(\frac{\partial w}{\partial x}\right)^2\left(\frac{\partial^2 w}{\partial x^2}\right)-\left(\frac{\partial w}{\partial y}\right)^2\left(\frac{\partial^2 w}{\partial y^2}\right)-\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\left(\frac{\partial^2 w}{\partial y^2}\right)-\frac{1}{2}\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial w}{\partial y}\right)^2-\frac{\partial^2 w}{\partial x\partial y}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}$$

For the stiffeners we use an analysis similar to that employed in another earlier reference[6]

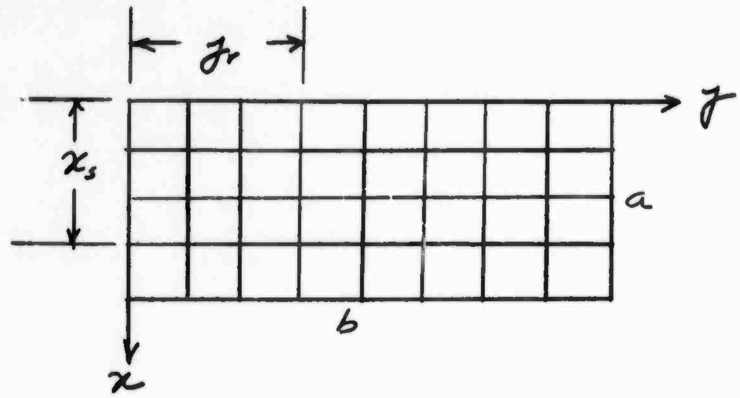


Fig. 10 Location of Stiffeners

For a stiffener located at $y = y_r$

$$V_r = \int_0^a \int_{A_r} \left[\frac{E}{2}(1-\lambda)\,\epsilon_x^2 + E\lambda e_s \epsilon_x \right] dA_r\,dx - \int_0^a \int_{A_r} \frac{E\lambda e_s^2}{2}\,dA_r\,dx \qquad [62]$$

Where A_r indicates integration over the cross sectional area of the rth stiffener.

For a stiffener located at $x = x_s$

$$V_s = \int_0^b \int_{A_s} \left[\frac{E}{2}(1-\lambda)\,\epsilon_y^2 + E\lambda e_s \epsilon_y \right] dA_s\,dy - \int_0^b \int_{A_s} \frac{E\lambda e_s^2}{2}\,dA_s\,dy \qquad [63]$$

Where A_s indicates integration over the cross sectional area of the s^{th} stiffener. Substituting the expressions for ϵ_x, ϵ_y

We obtain

$$\begin{aligned} V_r = \int_0^a \int_{A_r} \Big\{ & \frac{E}{2}(1-\lambda)\left[\frac{1}{4}\left(\frac{\partial w}{\partial x}\right)^4 - \left(\frac{\partial w}{\partial x}\right)^2 z \frac{\partial^2 w}{\partial x^2} + z^2\left(\frac{\partial^2 w}{\partial x^2}\right)^2 \right] \\ & + E\lambda e_s \left[\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - z\frac{\partial^2 w}{\partial x^2} \right] \Big\}\, dA_r\,dx \\ & - \int_0^a \int_{A_r} \frac{E\lambda e_s^2}{2}\,dA_r\,dx \end{aligned} \qquad [64]$$

Now

$$\int_{A_r} z^2\,dA_r = I_{A_r}$$ = the moment of inertia of the stiffener about its own neutral axis [65]

$$\int_{A_r} z\,dA_r = z_c A_r$$ where z_c is the distance from the neutral plane of the stiffener to the centroid of the stiffener (= 0)

Thus

$$\begin{aligned} V_r = \int_0^a \Big\{ & \frac{E}{2}(1-\lambda) A_r \frac{1}{4}\left(\frac{\partial w}{\partial x}\right)^4 + \frac{E}{2}(1-\lambda) I_{A_r}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 \\ & + E\lambda e_s A_r \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \Big\} dx - \frac{E\lambda e_s^2}{2} A_r a \end{aligned} \qquad [66]$$

The terms containing Ar are the contribution from stretching and the one containing I_{A_r} is the contribution from bending.

Similarly

$$V_s = \int_0^b \left\{ \frac{E}{2}(1-\lambda) A_s \frac{1}{4}\left(\frac{\partial w}{\partial y}\right)^4 + \frac{E}{2}(1-\lambda) I_{A_s}\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + E\lambda e_s A_s \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \right\} dy - \frac{E\lambda e_s^2}{2} A_s b \qquad [67]$$

These values of the work done and an appropriate series for the assumed deflection can then be used with the variational principle or energy approach to determine the plastic behavior of the plate. A particularly appropriate series for the rectangular plate would be

$$w = \sum_m \sum_n w_{omn}(t) \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \qquad [68]$$

This particular series can then be substituted into the internal work expression, V, above and the variational principle as given by equation [14] can be employed to determine the set of w_{omn}'s (in this case mn takes the place of n in the general equation given by equation [23]). In the cases involving pulses whose duration are greater than the fundamental period of the plate we can represent w as a single term series

$$w = w_o(t) \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} \qquad [69]$$

3. An alternate approximation for large plastic deformations

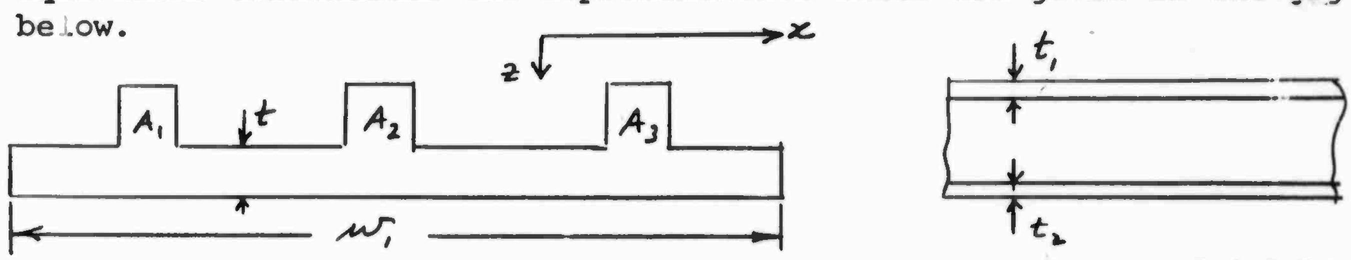
As is seen in the previous sections the elastic region is characterized by bending only (eq. [46] assumes bending stresses only) and the plastic region is described by equations involving both bending and membrane deformation. The large plastic deformation of plates can be predicted by using only membrane deformation.[29,30] The equation describing the behavior of an isotropic, cross stiffened or sandwich plate can be written as follows:[6]

$$T_x \frac{\partial^2 w}{\partial x^2} + T_y \frac{\partial^2 w}{\partial y^2} = \mu \frac{\partial^2 w}{\partial t^2} - P(x,y,t) \qquad [70]$$

where

$$T_x = \sigma_u h_x \qquad T_y = \sigma_u h_y$$

σ_u is the ultimate stress of the material and h_x, h_y are the equivalent thicknesses for stretching in the x and y directions respectively. The equivalent thicknesses for representative cases are given in the figure below.



Case A. Stiffened Plate

Case B. Sandwich Plate

$$h_x = \frac{A_1 + A_2 + A_3 + t w_1}{w_1}$$

$$h_x = t_1 + t_2$$

(A refers to cross sectional areas of stiffeners)

Fig. 11 Equivalent Thicknesses of Plating

For an isotropic plate the equivalent thickness is the actual thickness of the plate. The behavior of isotropic membranes under dynamic loading was studied by Baker and Hoffman[30] some years ago and by this author[29] more recently. The solution of equation [70] for a rectangular plate which has its edges supported can be written

$$w = \sum_m \sum_n q_{mn}(t) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \qquad [71]$$

Substituting into the equation of motions multiplying both sides of the equation by $\sin \frac{p\pi x}{a} \sin \frac{q\pi y}{b}$ and integrating over the area of the plate we find that $q_{mn}(t)$ satisfies the following equation

$$\ddot{q}_{mn} + \left(\frac{T_x}{\mu} \frac{m^2\pi^2}{a^2} + \frac{T_y}{\mu} \frac{n^2\pi^2}{b^2} \right) q_{mn} = \frac{4}{\mu ab} \int_0^a \int_0^b P(x,y,t) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\, dx\, dy \qquad [72]$$

The solution can be written in the same form as the elastic solution given in the previous section of this report, i.e.

$$w(x,y,t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[\sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \right] \times \frac{4}{\mu ab} \left[\frac{\int_0^a \int_0^b G(x,y) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\, dx\, dy}{p_{mn}^2} \right] \left[p_{mn} \int_0^t f(\tau) \sin p_{mn}(t-\tau)\, d\tau \right] \qquad [73]$$

where the load distribution $P(x,y,t)$ was represented as follows:

$$P(x,y,t) = G(x,y)\, f(t) \qquad [74]$$

and p_{mn} is

$$p_{mn} = \sqrt{\frac{T_x}{\mu} \frac{m^2\pi^2}{a^2} + \frac{T_y}{\mu} \frac{n^2\pi^2}{b^2}} \qquad [75]$$

4. Fitting the plate theories together

The previous sections have given various theories for describing the regimes of plate deformation. These theories can be used in conjunction with each other to follow the complete behavior of the plate from the elastic through the plastic region. The procedure is as follows:

a. Compute the elastic behavior up to the point where yield starts, then

b. Using the initial conditions determined from the elastic regime and the plastic relations given in 2 or 3, determine the plastic deformation until the velocity of the plate reaches zero for the first time. It will be assumed that the plate is "frozen" at this time in the same manner that Allen[25] used in the beam case. The final plastic deformation is then given by this frozen configuration.

C. Cylindrical shell type structures

The theory of plastic deformation of cylindrical shells under short duration impulsive loads has been given in a series of reports by the present author.[9,10] The general behavior was described in the earlier sections of this report. The energy absorbed or work done by the internal forces during deformation is as follows:

$$V = \frac{E(1-\lambda)taL}{2(1-\nu^2)}\int_0^1\int_0^{2\pi}\bar{\alpha}\,dx'd\phi + \frac{E(1-\lambda)taL}{6(1-\nu^2)}\int_0^1\int_0^{2\pi}\bar{\beta}\,dx'd\phi$$

$$+\frac{\lambda E e_s taL}{\sqrt{3}}\int_0^1\int_0^{2\pi}\left\{\left[\frac{(2\bar{\beta}+\bar{\gamma})\sqrt{\bar{\alpha}+\bar{\gamma}+\bar{\beta}}}{4\bar{\beta}} + \frac{(4\bar{\alpha}\bar{\beta}-\bar{\gamma}^2)}{8\bar{\beta}\sqrt{\bar{\beta}}}\sinh^{-1}\left(\frac{2\bar{\beta}+\bar{\gamma}}{\sqrt{4\bar{\alpha}\bar{\beta}-\bar{\gamma}^2}}\right)\right]\right. \qquad [76]$$

$$\left.-\left[\frac{(-2\bar{\beta}+\bar{\gamma})\sqrt{\bar{\alpha}-\bar{\gamma}+\bar{\beta}}}{4\bar{\beta}} + \frac{(4\bar{\alpha}\bar{\beta}-\bar{\gamma}^2)}{8\bar{\beta}\sqrt{\bar{\beta}}}\sinh^{-1}\left(\frac{-2\bar{\beta}+\bar{\gamma}}{\sqrt{4\bar{\alpha}\bar{\beta}-\bar{\gamma}^2}}\right)\right]\right\}$$

where

$$-E\lambda e_s^2\pi aLt$$

$$\bar{\alpha}(x',\phi) = \left(\frac{w_0}{a}\right)^4\left(\frac{a}{L}\right)^4\frac{1}{4}\left(\frac{\partial f}{\partial x'}\right)^4 + \left(\frac{w_0}{a}\right)^4\left(\frac{a}{L}\right)^2\frac{1}{2}\left(\frac{\partial f}{\partial x'}\right)^2\left(\frac{\partial f}{\partial \phi}\right)^2 - \nu\left(\frac{w_0}{a}\right)^3\left(\frac{a}{L}\right)^2 f\left(\frac{\partial f}{\partial x'}\right)^2$$
$$+\frac{1}{4}\left(\frac{w_0}{a}\right)^4\left(\frac{\partial f}{\partial \phi}\right)^4 - \left(\frac{w_0}{a}\right)^3 f\left(\frac{\partial f}{\partial \phi}\right)^2 + \left(\frac{w_0}{a}\right)^2 f^2$$

$$\bar{\gamma}(x',\phi) = -\left(\frac{w_0}{a}\right)^3\left(\frac{a}{L}\right)^4\frac{t}{D}\left(\frac{\partial f}{\partial x'}\right)^2\left(\frac{\partial^2 f}{\partial x'^2}\right) - \left(\frac{w_0}{a}\right)^3\frac{t}{D}\left(\frac{\partial^2 f}{\partial \phi^2}\right)\left(\frac{\partial f}{\partial \phi}\right)^2$$
$$+2\left(\frac{w_0}{a}\right)^2\frac{t}{D}\frac{\partial^2 f}{\partial \phi^2}f - \nu\left(\frac{w_0}{a}\right)^3\left(\frac{a}{L}\right)^2\frac{t}{D}\left(\frac{\partial f}{\partial x'}\right)^2\left(\frac{\partial^2 f}{\partial \phi^2}\right)$$
$$-\nu\left(\frac{w_0}{a}\right)^3\left(\frac{a}{L}\right)^2\frac{t}{D}\left(\frac{\partial^2 f}{\partial x'^2}\right)\left(\frac{\partial f}{\partial \phi}\right)^2 + 2\nu\left(\frac{w_0}{a}\right)^2\left(\frac{a}{L}\right)^2\frac{t}{D}f\left(\frac{\partial^2 f}{\partial x'^2}\right)$$
$$-2(1-\nu)\left(\frac{w_0}{a}\right)^3\left(\frac{a}{L}\right)^2\frac{t}{D}\left(\frac{\partial^2 f}{\partial x'\partial \phi}\right)\left(\frac{\partial f}{\partial x'}\right)\left(\frac{\partial f}{\partial \phi}\right) \qquad [77]$$

$$\bar{\beta}(x',\phi) = \left(\frac{w_0}{a}\right)^2\left(\frac{a}{L}\right)^4\left(\frac{t}{D}\right)^2\left(\frac{\partial^2 f}{\partial x'^2}\right)^2 + 2\nu\left(\frac{w_0}{a}\right)^2\left(\frac{t}{D}\right)^2\left(\frac{a}{L}\right)^2\left(\frac{\partial^2 f}{\partial x'^2}\right)\left(\frac{\partial^2 f}{\partial \phi^2}\right)$$
$$+\left(\frac{w_0}{a}\right)^2\left(\frac{t}{D}\right)^2\left(\frac{\partial^2 f}{\partial \phi^2}\right)^2 + 2(1-\nu)\left(\frac{w_0}{a}\right)^2\left(\frac{a}{L}\right)^2\left(\frac{t}{D}\right)^2\left(\frac{\partial^2 f}{\partial x'\partial \phi}\right)^2$$

In the above relations a linear hardening material was assumed as was done in the case of plates in previous sections of this report and it was assumed that the deflection took on a pattern $w = w_0 f(x',\phi)$ where $x' = x/L$. In these equations a is the radius of the shell, L the length, t the thickness, ν Poisson's ratio .

For collapse type behavior where the shell forms a hinge line straight across the center of the shell on the side facing the blast, the deformation pattern is described as follows:

$$w(x',\phi) = a\cos\phi - \sqrt{a^2 - \frac{d_0^2}{4}[1-2x']^2} \quad \text{for} \quad 0<x'<\tfrac{1}{2} \qquad [78]$$

$$= a\cos\phi - \sqrt{a^2 - \frac{d_0^2}{4}[1+2x']^2} \quad \text{for} \quad -\tfrac{1}{2}<x'<0$$

For buckling the pattern is as follows

$$w(x',\phi) = w_0 \sin\pi x' \, e^{-k\phi}\cos n\phi \quad \text{for} \quad 0<\phi<\pi \qquad [79]$$

$$= w_0 \sin\pi x' \, e^{-k(2\pi-\phi)}\cos n(2\pi-\phi) \quad \text{for} \quad \pi<\phi<2\pi$$

and finally for a very short terms impulse where the shell is loaded over half the circumference and a "ring type" pattern results, we have

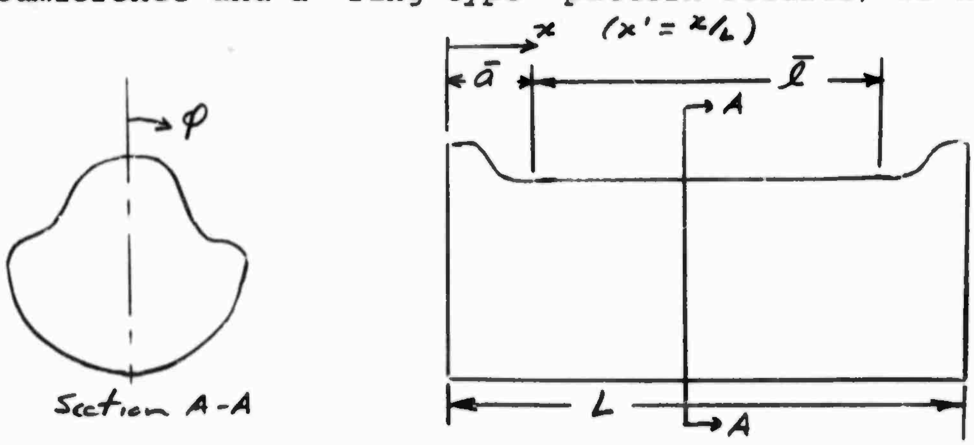


Fig. 12 Deformation Pattern of Shell with Sprayed Explosive Over Half the Circumference

$$w = w_0 \tfrac{1}{2}\left(1 - \cos\frac{\pi x'}{\bar{a}/L}\right)\sin^2 2\phi \qquad x'<a/L, \ \phi<\pi/2 \qquad [80]$$

$$= w_0 \sin^2 2\phi \qquad \bar{a}/L < x' < (\bar{a}/L + \bar{l}), \ \phi < \pi/2$$

by symmetry we double the energy obtained for the first quadrant ($\phi < \pi/2$); also the energy for $(\bar{l} + \bar{a}/L) < x' < L$ is the same as that for $x' < a/L$ (also by symmetry).

The above functions have been programmed for the computer. Some numerical results are given in previous reports[9,10] and some further results and comparison with experiments will be given in the next section of this report.

VII. A more exact theory for axially symmetric elastic-plastic deformation of cylindrical shells

A. Uniform isotropic shells

The equation of elastic deformation of isotropic cylindrical shells under axisymmetric loading is

$$D\frac{\partial^4 w}{\partial x^4} + \frac{Eh}{a^2} w + \rho h \frac{\partial^2 w}{\partial t^2} = p(x,t) \qquad [81]$$

where $D = \frac{Eh^3}{12(1-\nu^2)}$, h = thickness, E = elastic modulus, ρ = mass density, $p(x,t)$ is the load distribution.

This equation can be solved subject to boundary conditions at the edges x = 0, ℓ and appropriate initial conditons. For a uniform shell made of perfectly plastic material, the shell becomes plastic when the yield condition* is reached at a given locations, i.e. when

$$\sigma_x^2 - \sigma_x \sigma_\varphi + \sigma_\varphi^2 = \sigma_0^2 \qquad (\tau_{x\varphi} = 0) \qquad [82]$$

If this condition occurs at the edge of a fixed <u>beam</u> then the boundary condition at the edge is

w = 0 (edge deflection zero)

$M = M_o$ (bending moment equal to plastic moment) [83]

If this plastic condition occurs at any other section then we must resort to continuity conditions for beams, i.e.

$w_L = w_R$ (deflection on the left of the hinge = deflection on the right of the hinge)

$M_L = M_R$ (left bending moment = right bending moment) [84]

$Q_L = Q_R$ (left shear = right shear)

$M_L = M_o$ (bending moment equal to plastic moment)

The finite difference computer program developed several years ago for beams[4] can be applied directly to this problem by making changes in the yield condition.* It is anticipated that the basic behavior of shells going into "ring type" deformation and center hinging can be ascertained directly from this theory.

B. Uniform anisotropic shells (including layered shells)

The basic behavior of certain anisotropic shells under general non-axisymmetric loading was given by Flugge[31] and applied to some dynamic problems by this author.[32] The equation for axially symmetric lateral deformation can be written down immediately from these results. This equation is the anisotropic counterpart of equation [81] and includes such factors as initial external or internal static

pressure. It is as follows:

$$a^4 \frac{\partial^4 w}{\partial x^4}\left[\frac{K_x}{D_x a^2} + \frac{K_x}{D_x a^2}\delta \frac{\left(\bar{\xi}\frac{K_x}{D_x a^2} + \frac{S_x}{aD_x}\right)}{1 - P'/D_x} - \frac{S_x}{aD_x}\rho \frac{\left(\bar{\xi}\frac{K_x}{a^2 D_x} + \frac{S_x}{aD_x}\right)}{1 - P'/D_x}\right] \tag{85}$$

$$+ a^2 \frac{\partial^2 w}{\partial x^2}\left[\frac{P'}{D_x} - \frac{S}{aD_x}2\nu + \frac{K_x}{a^2 D_x}\delta \frac{\left(-\frac{p'a}{D_x} - \frac{D_\nu}{D_x}\right)}{1 - P'/D_x} - \frac{S_x}{aD_x}\left(-\frac{p'a}{D_x} - \frac{D_\nu}{D_x}\right) + \left(\frac{p'a}{D_x} + \frac{D_\nu}{D_x}\right)\left(\frac{\bar{\xi}\frac{K_x}{a^2 D_x} + \frac{S_x}{aD_x}}{1 - P'/D_x}\right)\right]$$

$$+ w\left[\bar{\xi}\frac{K_\phi}{a^2 D_x} - \alpha\frac{S_\phi}{aD_x} + \frac{D_\phi}{D_x} + \left(\frac{p'a}{D_x} + \frac{D_\nu}{D_x}\right)\frac{\left(-\frac{p'a}{D_x} - \frac{D_\nu}{D_x}\right)}{1 - P'/D_x}\right]$$

$$= -\frac{\mu a^2}{D_x}\frac{\partial^2 w}{\partial t^2} + \frac{a^2 p_r(x,t)}{D_x}$$

p' is the initial pressure in load/unit area, P' is the edge loading in load/unit length, μ is the mass per unit area, $p_r(x,t)$ is the radial pressure. The appropriate values of the constants for sandwich, cross stiffened, and orthotropic shells are given in the previously cited reference.[32] One case of considerable interest is the layered shell which is not given specifically in the above reference[32] but the constants can be written down directly by using the work of Flugge[31] and this reference.[32] First assume that each layer of the n layers is orthotropic and has a stress-strain law

$$\begin{aligned} \sigma_{xn} &= E_{xn}\epsilon_x + E_{\nu n}\epsilon_\phi \\ \sigma_{\phi n} &= E_{\nu n}\epsilon_x + E_{\phi n}\epsilon_\phi \\ \tau_{x\phi n} &= G_n \gamma_{x\phi} \end{aligned} \tag{86}$$

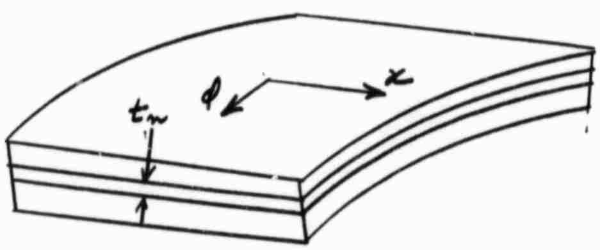


Fig. 13 Layered Shell

Then

$D_x = \sum_n E_{xn} t_n$ where E_{xn} is the elastic modulus in the x direction of the nth layer and t_n is the thickness of the layer. [87]

$D_\varphi = \sum_n E_{\varphi n} t_n$ where $E_{\varphi n}$ is the elastic modulus in the φ direction of the nth layer [88]

$$D_\nu = \sum_n E_{\nu n} t_n \quad [89]$$

$$K_x = \frac{1}{12} \sum_n E_{xn} t_n^3 + \sum_n E_{xn} t_n r_n^2 \quad [90]$$

where r_n is the distance from the midsurface of the entire shell to the midsurface of the nth layer

$$K_\varphi = \frac{1}{12} \sum_n E_{\varphi n} t_n^3 + \sum_n E_{\varphi n} t_n r_n^2 \quad [91]$$

$$S_\varphi = S = S_x = 0 \quad [92]$$

and the other constants are

$$\bar{\rho} = 1,\ \bar{\xi} = 1,\ \beta = 1,\ \bar{\eta} = 1,\ \gamma = 1,\ \bar{\epsilon} = 1,\ \rho = 0,\ \delta = -1$$
$$\xi = 1,\ \bar{\alpha} = 1,\ \bar{\beta} = 1,\ \bar{\delta} = 2,\ \bar{\gamma} = -1,\ \alpha = 0 \quad [93]$$

VIII. Some simple applications and comparisons with experiment

A. Beams

Consider the simple case of the uinform cantilever under blast from pentolite and let us apply the impulse theory, neglecting elastic energy. As given earlier in the report the energy flux (energy per unit area) can be written in terms of the impulse per unit area as follows:

$$\bar{E}_f = \frac{\bar{I}^2}{2\mu} \qquad \begin{array}{l} \bar{I} = \text{impulse per unit area applied to beam} \\ \mu = \text{mass per unit area of beam} \end{array} \quad [94]$$

The value which Goodman[18] obtained for the side on impulse per unit area was given in a previous section of this report and its value is

$$\frac{\bar{I}}{p_0^{2/3} W^{1/3}} = \frac{478.7}{p_0^{1/3} X} - \frac{3593}{(p_0^{1/3} X)^2} \qquad \frac{\text{psi millisec}}{\text{lb}^{1/3}} \quad [95]$$

where $X = Z/.1323$

p_0 = atmospheric pressure in ATM

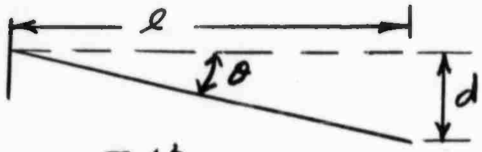
$Z = R/W^{1/3}$ R = distance from explosion to target

W = weight of explosive

Applying the previously developed ideas of plastic deformation of hinging cantilever beams, the energy absorbed at the hinge is

$$V = \int_0^\theta M(\theta)\, d\theta \quad [96]$$

Assume a constant hinge moment M_o (perfectly plastic case). We obtain



$$V = M_0 \theta$$

$$\theta = d/\ell$$

Thus

$$d/\ell = \bar{E}_f A / M_0 \qquad M_0 = \frac{\sigma_0 t^2 w}{4} \qquad [97]$$

where w = width of beam

σ_0 = yield stress

t = thickness of beam

A = loaded area of beam

Using the appropriate values for the energy flux, the impulse, etc. we obtain

$$d = \frac{2\left[\frac{478.7 \times .1323}{z} - \frac{3593 \times \overline{.1323}^2}{z^2}\right] \ell^2 W^{2/3}}{\rho \sigma_0 t^3} \qquad [98]$$

ρ = mass density of beam material

Consider an aluminum beam 12" long, .091" thick and 1" wide. The results for the tip deflection, d, given by the above relation and the corresponding experimental results[16] are given in the following table

Table I. Theoretical and Experimental Results for Cantilevers

Charge Weight	Distance	Theoretical Tip Deflection	Experimental Tip Deflection
½ #	2.5'	9"	9.4"
½ #	3'	7.1"	7.3"
½ #	4'	4.7"	2.7"
½ #	5'	3.3"	1"
3 #	14'	5.3"	.05"

There are sufficient calculations shown in the table to illustrate the trend. For small charges very close to the beam, i.e. for very short duration pulses where diffraction loading is of primary consideration, the impulse theory coupled with the rigid plastic assumption predicts good results. However for large charges further away where gust loading has a substantial contribution the results predicted by impulse theory and rigid plastic behavior are not good. Therefore recourse must be made to the more complete theory such as the finite difference approach mentioned earlier.

B. Cylindrical shells

1. Shells subjected to blast at various ranges

a. Prediction using energy formula directly

We assume perfectly plastic behavior of the material. Thus the curves of Fig. 8 of the earlier reference[10] can be used directly. For completeness these curves are reproduced here in Fig. 14. Some examples are given below which illustrate the application of the theory.

(1) A charge weight of 389# of pentolite is exploded 29' away from a steel shell; the shell is 3"OD, 6" long, .019" thick and goes into a collapse pattern. The problem is to predict the final deformation. The experiment showed $d_0/D \approx 1.0$

The energy flux predicted by the formula given in an earlier section of this report is

$$\bar{E}_f = 19.2\times10^3 \left(\frac{W^{5/4}}{R^{11/4}}\right) \frac{\# \, in}{in^2} = 3150 \frac{\# \, in}{in^2}$$

The total energy from the explosion will be

$$V = \bar{E}_f \times A_p \quad (A_p = \text{projected area of shell} = LD)$$

$$V = 3150 \times 6 \times 3 = 56{,}700 \ \# \, in$$

The theory says that this energy is absorbed in plastically deforming the shell. Since we are assuming a perfectly plastic material, assume that σ_s is the ultimate stress, which was found experimentally to be about 50,000 psi.

Thus we obtain

$$\bar{V} = \frac{V}{\frac{\sigma_s t a L}{\sqrt{3}}} = \frac{56{,}700 \times \sqrt{3}}{50\times10^3 \times .019 \times 1.5 \times 6} \approx 11$$

Going into the curve, a $\bar{V} = 11$ corresponds to a $d_0/D >> 1$ for collapse of a shell with $L/D = 2$. This means that the theory predicts deflections which are much too high.

(2) Next consider a charge weight of 8.4# at 8' away. The shell is steel, 3"OD, 11.62" long, .019" thick (collapse pattern). The experiment showed $d_0/D \approx .7$

The energy flux predicted by the formula is

$$\bar{E}_f = 19.2\times10^3 \left(\frac{8.4^{5/4}}{8^{11/4}}\right) = .9\times10^3 \ \frac{\# \, in}{in^2}$$

Thus

$$V = 31{,}500 \ \# \, in$$

$$\bar{V} = (31{,}500 \times 1.732)/(50\times10^3 \times .019 \times 1.5 \times 11.62) = 3.3$$

Even in this case a $\bar{V}$ of 3.3 predicts collapse deformation which is much too great.

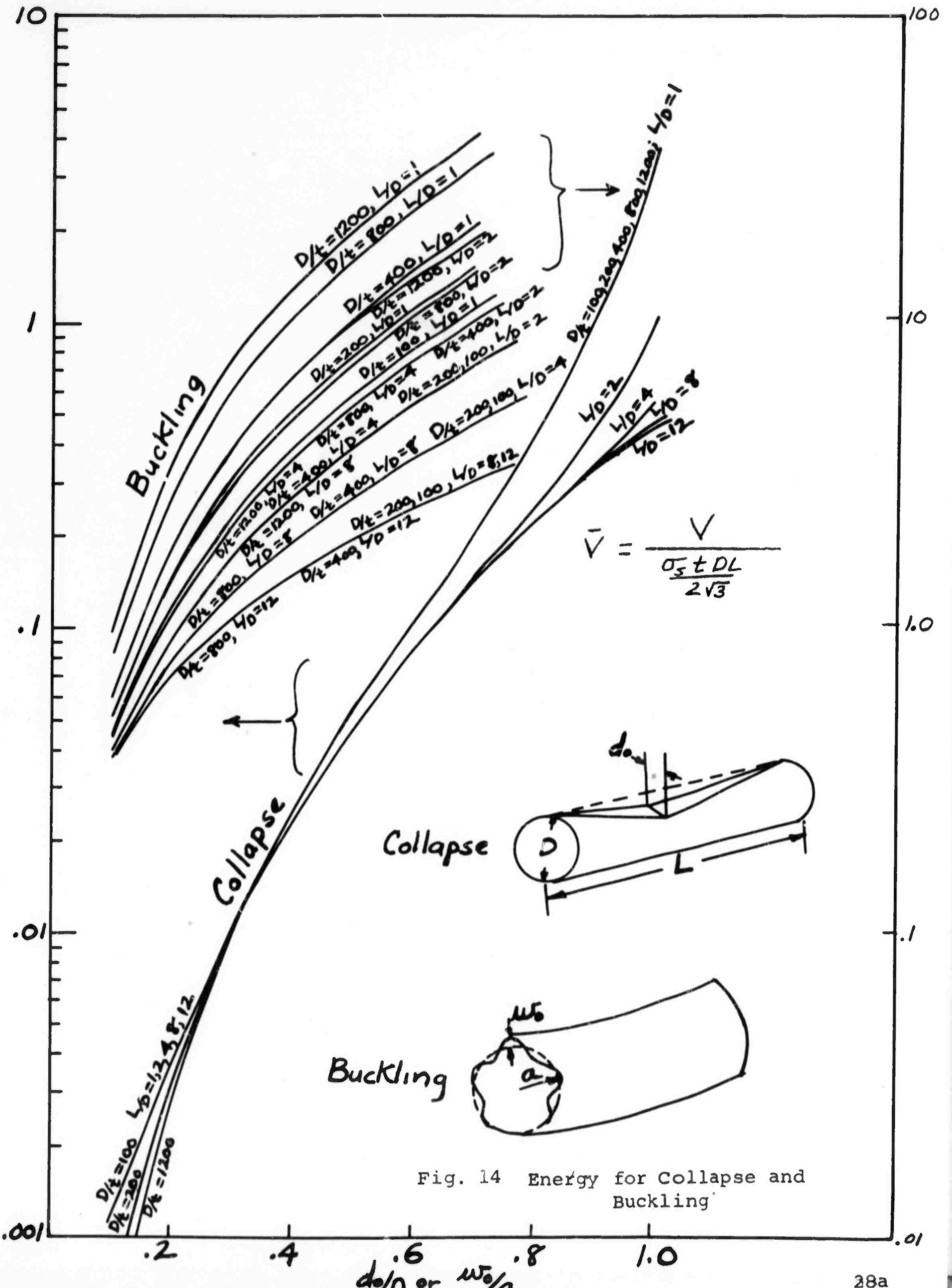


Fig. 14 Energy for Collapse and Buckling

(3) Next consider 1.1# at 3' away from the steel shell 3" OD, 11.62" long, .019" thick with a collapse pattern. The experiment showed $d_0/D \approx .5$

$$\bar{E}_f = 1.06 \times 10^3 \frac{\# in}{in^2}, \quad \bar{V} = 3.9$$

Again the prediction is too large.

Prediction using side on impulse to compute energy

Instead of working with the energy formula supposing we use the side on impulse[1,2,18] to compute the energy. The energy will then be given by

$$\bar{E}_f = \frac{1}{2} \frac{(\bar{I})^2}{\mu}$$

(1) Consider the first shell solved in the previous section. For this case the side on impulse, $\bar{I}$, is about 90 psi millisec, thus

$$\bar{E}_f = \frac{.5 \times (.09)^2 \times 386}{.284 \times .019} = 290 \frac{\# in}{in^2}, \quad V = 5200 \# in$$

Thus

$$\bar{V} = \frac{5200 \times 1.732}{50 \times 10^3 \times .019 \times 9} \approx 1$$

This value of $\bar{V}$ predicts a deformation of $d_0/D \approx 1$; a deformation of $d_0/D \approx 1$ was found experimentally.

(2) Consider the second shell solved in the previous section. For this case the side on impulse is about 25 psi millisec

$$\bar{E}_f = \frac{.5 \times (.025)^2 \times 386}{.284 \times .019} = 22.4 \frac{\# in}{in^2}$$

$$V = 22.4 \times 3 \times 11.62 = 780 \# in$$

$$\bar{V} = (780 \times 1.732)/(50 \times .019 \times 1.5 \times 11.62) \approx .08$$

Coming over on the curve of $L/D = 4$ we obtain a $d_0/D \approx .6$
The experiment gave a value of $d_0/D \approx .7$

(3) For the third shell $\bar{I}$ = 14.5 psi millisec, $\bar{E}_f$ 8.1# in/in², $\bar{V} \approx .022$
Thus d_0/D for the theory is .4. Experiment showed $d_0/D \approx .5$

(4) Consider another case of a steel shell exposed to 8.4# explosive at 5.8'. The shell is 6" diameter, 17.5" long and .035" thick and goes into a collapse pattern. The side on impulse is about 30 psi millisec.

The $\bar{V}$ for this case is about .03 which corresponds to a $d_0/D \approx .4$
The experimental value of d_0/D was about .5.

(5) Now consider buckling failure of an aluminum shell. The shell is 3"OD, 2" long, .008" thick, subject to an explosion of 1# at a distance of 8'. We use an experimentally determined σ_s of about 15000 psi. The assumed value of k in the buckling formula [79] is k = ¼.
These values were assumed for all the subsequent buckling calculations in this report.

The side on impulse for this case is about 8psi millisec. The calculated value of $\bar{V}$ is about .4. Going into the curve we find that for buckling the value of w_0/a is about .05. The experimental value of w_0/a for this test was found to be .08.

(6) Next consider buckling of an aluminum shell 3"OD, 5" long and .008" thick subject to an explosion of 1.1# at 10'. The side on impulse is about 6 psi millisec. The calculated value of $\bar{V}$ is about .3 which gives a w_0/a of about .07. The experimental value was about .1.

(70 Lastly consider collapse of an aluminum shell 3"OD, 9" long, and .042" thick subjected to an explosion of 115# at 17'. The σ_s for this case was 50,000 psi. The side on impulse is about 64 psi millisec. The calculated $\bar{V}$ was about .31 giving a $d_0/D \approx .8$. The experimental value of d_0/D was about .6.

2. Sprayed explosive

In section IIB 3 of this report we discussed the behavior of shells under very short duration loading such as would be obtained from sprayed explosive. It has been found experimentally that the fixed ended shell deforms into a pattern which corresponds to an abrupt change at the ends and an almost uniform deformation along the length. Consider as a first approximation that end effects are negligible. The energy of deformation can then be written as follows: (assuming perfectly plastic behavior)

$$V \approx \frac{\sigma_s t a L}{\sqrt{3}}\left[\frac{w_0}{a}\, 2\int_0^{2\pi} f\,d\phi - \left(\frac{w_0}{a}\right)^2 \int_0^{2\pi}\left(\frac{df}{d\phi}\right)^2 d\phi\right]_{f=\sin^2 2\phi} \qquad [99]$$

Now for a uniform impulse over half the cylinder

$$\bar{I} = I_0 \rho t = \sqrt{V(2\rho t)/\pi a L} \qquad [100]$$

Evaluating the integrals contained in V above, we obtain

$$\bar{I} = \sqrt{\frac{\sigma_s t^2 \rho}{\sqrt{3}}\left(\frac{w_0}{a} - \frac{1}{2}\frac{w_0^2}{a^2}\right)} \qquad [101]$$

An aluminum cylinder was tested, the parameters of which were

$$a = 1.5'' , \quad \sigma_s = 30{,}000 \text{ psi}$$
$$\rho = .1^{\#}/\text{in}^3 , \quad t = .022''$$

The experimental deformation was about ½" under a measured impulse of about 27 psi millisec.

Using the above relation we find that for $w_0/a = .33$ (which corresponds to ½" deflection in a cylinder with a = 1.5")

$$\bar{I} \approx 24 \text{ psi millisec}$$

Thus the theory predicts reasonably well the impulse for a given deflection for this particular test.

C. Cpnclusions based upon experimental comparisons

It is shown that the side on impulse criterion gives excellent results for beam deformation in the case of very small charges and short distance. For larger charges and distances we will have to resort to the more exact elastic-plastic theory using the differential equation.

For cylindrical shells it seems that the side on impulse gives excellent results for rigid-plastic collapse and buckling behavior of all the shells under long or short duration blasts. When more cases of sprayed explosive results become available, they will be compared with theory.

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13. ABSTRACT

This report discusses the general types of failures of typical structures that are used in aircraft and missiles. The theories of elastic and plastic deformation of these structures are presented and comparison with experiments of simple structures is given. It is found that the side on impulse can be used together with rigid-plastic theory of buckled and collapsed cylinders to predict plastic deformation for a wide range of pulse durations. For beam type structures the impulse and rigid plastic approximations seem to only hold for pulses of very short duration.

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